

## Forecasting Tool for K–12 Teacher Quarantine: Technical Supplement

February 2021

The purpose of the *Forecasting Tool for K–12 Teacher Quarantine* is to help districts and schools forecast the percentage of teachers who must stay at home on a typical day as a result of having or being exposed to COVID-19. This technical supplement discusses the analytic framework for the tool’s calculations.

The analytic framework first considers teachers who are not fully vaccinated and develops a forecast for the percentage of those teachers who must stay at home. It then adapts those calculations for fully vaccinated teachers. Finally, it combines the two results to develop a forecast for the overall percentage of teachers who must stay at home. Table 1 defines key terms used in this framework.

**Table 1. Key terms**

Term	Definition
Asymptomatic case	A person who tests positive for COVID-19 but will never develop symptoms
Close contact	An interaction in which two people are close enough for a sufficient time such that if one person had COVID-19 during that interaction, district rules would require the other person to enter quarantine. Under guidelines from the Centers for Disease Control and Prevention (CDC), being within six feet of another person for a total of 15 minutes within a 24-hour period counts as a close contact (CDC 2020c). Districts may also classify additional interactions as close contacts, such as being in the same classroom for an extended time.
Fully vaccinated person	A person for whom at least two weeks have elapsed since receiving the second dose of a two-dose COVID-19 vaccine or one dose of a single-dose vaccine (CDC 2021c)
Infected person	A person who is within the contagious period for COVID-19 and has either tested positive or will test positive for COVID-19. According to the CDC, the contagious period starts 48 hours before the onset of symptoms or, for asymptomatic cases, 48 hours before a specimen was taken for a positive test (CDC 2020c). The contagious period ends 10 days after the onset of symptoms and 24 hours after resolution of fever or, for asymptomatic cases, 10 days after a positive test (CDC 2020b).
Isolation	Staying at home because of having tested positive for COVID-19
Quarantine	Staying at home because of having had close contact with someone else who tested positive for COVID-19
Symptomatic case	A person who tests positive for COVID-19 and either has developed or will eventually develop symptoms
Symptomatic rate	Number of symptomatic cases as a fraction of all reported COVID-19 cases

Note: Throughout this document, testing “positive for COVID-19” is used as a shorthand for testing positive for the virus that causes COVID-19, SARS-CoV-2.

### A. Forecasts for teachers who are not fully vaccinated

Two types of events would make a teacher unavailable to come to school. First, based on recommendations from the Centers for Disease Control and Prevention (CDC), the teacher would need to

enter a 14-day quarantine after coming into close contact with an infected person (CDC 2021d). Second, the teacher would need to enter isolation after receiving a positive COVID-19 test, with the isolation period lasting 10 days after the onset of symptoms or 10 days after the positive test (if asymptomatic) (CDC 2020b).

Based on these quarantine and isolation periods, the teacher would need to stay at home on the current day if the teacher:

- Had close contact with an infected person within the past 14 days (because the teacher would still be within the required quarantine period); or
- Received a positive COVID-19 test within the past 10 days (because the teacher would still be within the required isolation period), assuming the onset of symptoms occurred around the same time as the test.

Because events within the past 14 days could affect whether a teacher needs to stay at home on the current day, this tool constructs a 14-day timeline leading up to the current day—with the current point in time being designated as the beginning of day 15—and calculates the likelihood of events in days 1 through 14 that could prevent the teacher from coming to school on day 15. The remainder of this section discusses the tool’s approach to calculating (1) the probability of coming into close contact with an infected person during those 14 days and (2) the probability of receiving a positive COVID-19 test during the most recent 10 days.

### 1. Probability of coming into close contact with an infected person on a single day

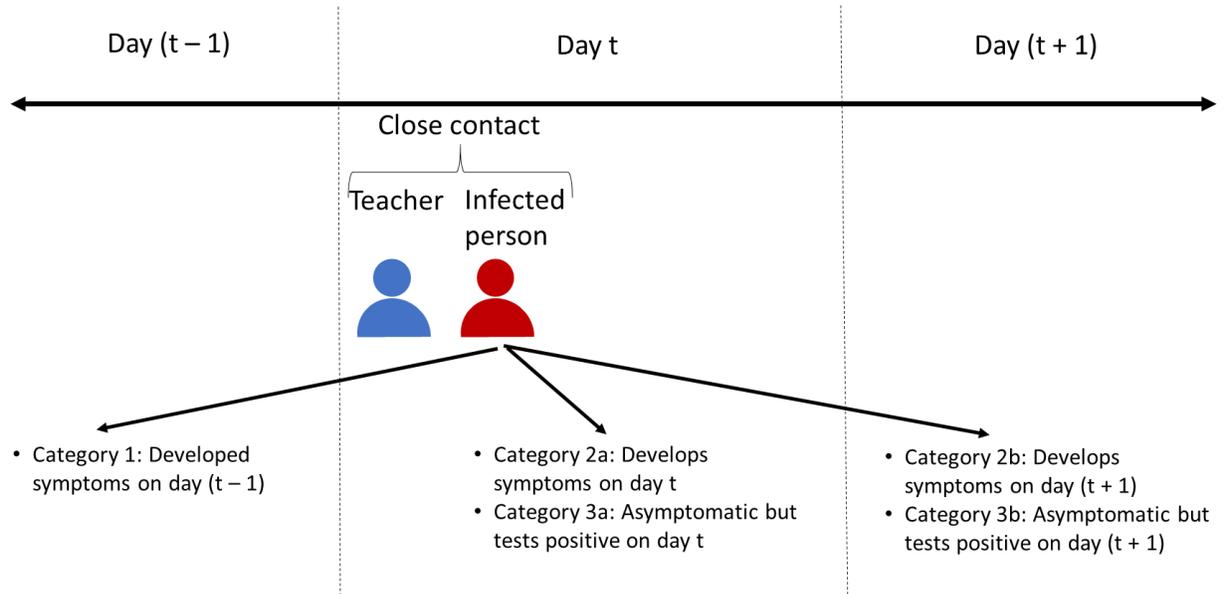
To calculate the probability of having come into close contact with an infected person anytime during the past 14 days, it is first necessary to calculate the probability of coming into close contact with an infected person *on a single day*. This will depend on (1) the probability that a single individual with whom the teacher has close contact is infected and (2) the number of individuals with whom the teacher has close contact during the course of a day.

Suppose a close contact occurs between the teacher and another person at the beginning of day  $t$ . This close contact will trigger quarantine if the other person belongs to any of the following three groups of infected people (Figure 1):

- Category 1: The other person developed symptoms on day  $(t - 1)$  and will eventually test positive for COVID-19. This tool assumes a degree of delayed precaution—specifically, that people who develop symptoms continue to go about their normal lives for one day before they take full precautions and self-isolate. Therefore, people who developed symptoms on day  $(t - 1)$  may still have close contact with the teacher on day  $t$  before they self-isolate. However, someone who developed symptoms on day  $(t - 2)$  or earlier will start self-isolating a day afterwards and, therefore, will not have any possibility of coming into close contact with the teacher on day  $t$ .
- Category 2: The other person will develop symptoms on day  $t$  (Category 2a) or day  $(t + 1)$  (Category 2b) and eventually test positive for COVID-19. This close contact occurs within 48 hours before the other person develops symptoms, a period in which the CDC regards this person as contagious.
- Category 3: The other person is asymptomatic but will test positive for COVID-19 on day  $t$  (Category 3a) or day  $(t + 1)$  (Category 3b). This close contact occurs within 48 hours before the other person tests positive for COVID-19, a period in which the CDC regards this person as contagious. This tool assumes that people who receive a positive test immediately self-isolate, so an infected person who

received a positive test before day  $t$  will not have any possibility of coming into close contact with the teacher on day  $t$ .

**Figure 1. Categories of infected people with whom a teacher could have close contact**



All of these categories of infected people refer to cases of COVID-19 that are confirmed with a positive test. A teacher's close contact with someone with an unreported case of COVID-19 would not trigger a quarantine, so long as the teacher does not later test positive for COVID-19. Therefore, this tool does not consider unreported cases of COVID-19.

The probability that the other person belongs to categories 1, 2a, or 2b depends on the daily number of people in the community who develop symptoms that eventually get confirmed with a positive test. This estimate is equal to the daily number of reported cases in the community multiplied by the symptomatic rate—the fraction of reported cases that are symptomatic—which the CDC estimates to be about 0.6 (CDC 2020a). For any specific individual, the timing of symptom onset (about 2 to 14 days after exposure) need not have a close connection with the timing of a test result. However, in the community as a whole, if the daily number of new reported cases is relatively stable (or can be reasonably approximated by an average during the period being considered), then the number of people who develop symptoms that get confirmed with a positive test should stabilize to about 60 percent of the daily number of new reported cases. For example, if a community typically had 10 new reported cases daily, but every day far more or far fewer than 6 people developed symptoms that eventually got confirmed with a positive test, then this scenario would be inconsistent with a 60 percent symptomatic rate.

Formally, define  $d$  as the average daily number of reported cases per 100,000 people in the community, and  $\sigma$  as the symptomatic rate of COVID-19. The daily probability that a randomly selected person in the community develops symptoms of COVID-19 (that eventually get confirmed with a positive test) is  $\sigma d/100000$ .

The probability of belonging to category 3a or 3b depends on the daily probability that a randomly selected person receives a positive test while asymptomatic. This probability is equal to  $(1 - \sigma)d/100000$ .

With these inputs, the tool considers three scenarios in which a close contact with a single other person on day  $t$  will force the teacher to enter quarantine:

- The other person developed symptoms on day  $(t - 1)$ —that is, belongs to category 1—which has a probability of  $\sigma d/100000$ .
- The other person will develop symptoms or receive an asymptomatic positive test on day  $t$ —that is, belongs to either category 2a or 3a—which has a probability of  $\frac{\sigma d}{100000} + \frac{(1-\sigma)d}{100000} = \frac{d}{100000}$ .
- The other person will develop symptoms or receive an asymptomatic positive test on day  $(t + 1)$ —that is, belongs to either category 2b or 3b—which has a probability of  $\frac{\sigma d}{100000} + \frac{(1-\sigma)d}{100000} = \frac{d}{100000}$ .

The probability that none of these scenarios occurs is  $(1 - \frac{\sigma d}{100000})(1 - \frac{d}{100000})^2$ . Therefore, the probability that a close contact with a single other person will force the teacher to enter quarantine is

$$(1) \quad c = 1 - (1 - \frac{\sigma d}{100000})(1 - \frac{d}{100000})^2.$$

Equation (1) applies to a close contact with a person from the general population, but certain groups, such as elementary and middle school students, are known to have lower rates of daily new COVID-19 cases (because they are less susceptible to getting COVID-19 and tend to interact more with other children, who are less likely to transmit COVID-19). Define  $\lambda$  as the ratio of daily new cases among elementary or middle school students to daily new cases in the general population. Adapting Equation (1), the probability that a close contact with a single other elementary or middle school student will force the teacher to enter quarantine is

$$(2) \quad \tilde{c} = 1 - (1 - \frac{\sigma \lambda d}{100000})(1 - \frac{\lambda d}{100000})^2.$$

Based on data from the COVID-19 School Response Dashboard from 10/12/2020 through 12/13/2020,  $\lambda = 0.52$  for elementary school students and  $\lambda = 0.62$  for middle school students (Oster et al. 2021). High school students demonstrate similar rates as the general population.

Finally, close contact with a third group, other school staff, will also be lower risk to the extent that this group has been vaccinated. Among school staff who have not been fully vaccinated, the daily number of new cases per 100,000 people will resemble the general population,  $d/100000$ . (This tool does not consider protective benefits from having received only the first dose of a two-dose vaccine series.) Among school staff who have been fully vaccinated, the daily number of new cases per 100,000 people will be  $(1 - e)d/100000$ , where  $e$  is the vaccine’s rate of effectiveness at preventing COVID-19, assumed to be 94 percent (the Moderna vaccine’s rate of effectiveness, about 1 percentage point lower than the Pfizer-BioNTech vaccine’s rate of effectiveness) (CDC 2021a, 2021b). Let  $v$  be the vaccination rate among teachers—the fraction of teachers who have been fully vaccinated. Adapting Equation (1), the probability that a close contact with a single other school staff member will force the teacher to enter quarantine is

$$(3) \quad c' = 1 - (1 - \frac{\sigma[(1-v)+v(1-e)]d}{100000})(1 - \frac{[(1-v)+v(1-e)]d}{100000})^2.$$

A teacher may have close contact with multiple people within a day. On day  $t$ , define  $A_t$  as the number of people from the community,  $F_t$  as the number of school staff, and  $S_t$  as the number of students with whom the teacher has close contact. The probability that none of the people with whom the teacher has close contact will force the teacher to enter quarantine is  $(1 - c)^{A_t}(1 - c')^{F_t}(1 - \tilde{c})^{S_t}$ . Therefore, on a single day  $t$ , the probability that at least one person with whom the teacher has close contact will force the teacher to enter quarantine is

$$(4) \quad Q_t = 1 - (1 - c)^{A_t}(1 - c')^{F_t}(1 - \tilde{c})^{S_t}.$$

## 2. Probability of coming into close contact with an infected person during the past 14 days

The prior discussion calculated the single-day probability of having to enter quarantine, but as discussed at the beginning of this document, the teacher will be in quarantine at the beginning of day 15 if the teacher had to enter quarantine during any of the days from 1 to 14. The probability of entering quarantine,  $Q_t$ , differs across three types of days within a week:

- *School days in which the teacher teaches at least some students in person.* On each of those days, the teacher will have close contact with  $A_t = a$  people from the community,  $F_t = f$  school staff, and  $S_t = s$  students. For ease of use, the tool treats  $a, f$ , and  $s$  as constants that do not vary across the school days in which the teacher teaches students in person. From Equation (4), the probability of entering quarantine on each of those days is  $Q_{in} = 1 - (1 - c)^a(1 - c')^f(1 - \tilde{c})^s$ .
- *School days in which the teacher teaches only remote students.* On each of those days, the teacher will have close contact with  $A_t = a$  people from the community and  $F_t = f$  school staff (assuming the teacher is working from a school building while teaching remote students). From Equation (4), the probability of entering quarantine on each of those days is  $Q_{vir} = 1 - (1 - c)^a(1 - c')^f$ .
- *Weekend days.* On each of those days, the teacher will have close contact with  $A_t = a$  people from the community. From Equation (4), the probability of entering quarantine on each of those days is  $Q_{end} = 1 - (1 - c)^a$ .

Each week consists of  $D_{in}$  days of in-person instruction,  $D_{vir}$  days of virtual instruction, and  $D_{end} = 7 - D_{in} - D_{vir}$  weekend days (or days that are effectively equivalent to weekend days because the teacher is not at work). The probability of avoiding the need to enter quarantine in the past 14 days is  $(1 - Q_{in})^{2D_{in}}(1 - Q_{vir})^{2D_{vir}}(1 - Q_{end})^{2D_{end}}$ . Therefore, the probability of entering quarantine at any point in the past 14 days is

$$(5) \quad U_Q = 1 - (1 - Q_{in})^{2D_{in}}(1 - Q_{vir})^{2D_{vir}}(1 - Q_{end})^{2D_{end}}.$$

The tool asks users to provide values for  $f, s, D_{in}$ , and  $D_{vir}$  based on the user's knowledge of the district's or school's educational approach.

Because it is likely difficult for a user to come up with a good estimate for  $a$ , the number of people from the community with whom a teacher has close contact on a typical day, the tool assumes a value for  $a$ . The assumed value is based on the Cuebiq Contact Index, which uses location data from a representative sample of more than 15 million cell phones to measure the number of times per day an average cell phone user is within 50 feet of another cell phone user from outside their household for at least five minutes (Cuebiq 2021). As of February 2021, the average cell phone user had about three such contacts per day. However, the Cuebiq Contact Index measures a broader set of contacts than those that the CDC defines as close contacts (being within six feet for 15 minutes). Therefore, this tool assumes that about half of the

contacts measured by the Cuebiq Contact Index are close contacts, setting  $a = 1.5$ . The forecasts generated by this tool are not highly sensitive to this assumption. For example, assuming that 20 or 80 percent of the contacts in this index are CDC-defined close contacts would lower or raise the final forecast from this tool typically by no more than 0.5 percentage points.

### 3. Probability of receiving a positive COVID-19 test in the past 10 days

Each day, another event that could make a teacher unavailable to come to school is that the teacher might receive a positive COVID-19 test and have to enter isolation. The single-day probability that the teacher has to enter isolation as a result of a positive COVID-19 test is

$$(6) \quad I = d/100000.$$

As discussed at the beginning of this document, the teacher will be in isolation at the beginning of day 15 if the teacher had to enter isolation during any of the most recent 10 days. The probability of avoiding the need to enter isolation over the past 10 days is  $(1 - I)^{10}$ . Therefore, the probability of entering isolation at any point in the past 10 days is

$$(7) \quad U_I = 1 - (1 - I)^{10}.$$

### 4. Probability of having to stay at home due to either quarantine or isolation

Combining the framework for quarantine and isolation, the teacher will have to stay at home at the beginning of day 15 if the teacher is either in quarantine or isolation. The probability of being in neither quarantine nor isolation is  $(1 - U_Q)(1 - U_I)$ . **Therefore, among teachers who are not fully vaccinated, the probability of having to stay home at the beginning of day 15 as a result of having entered quarantine in the past 14 days or entered isolation in the past 10 days is**

$$(8) \quad \begin{aligned} U_{notvac} &= 1 - (1 - U_Q)(1 - U_I) \\ &= 1 - (1 - Q_{in})^{2D_{in}}(1 - Q_{vir})^{2D_{vir}}(1 - Q_{end})^{2D_{end}}(1 - I)^{10}. \end{aligned}$$

## B. Forecasts for teachers who are fully vaccinated

The framework for teachers who are not fully vaccinated also applies to teachers who are fully vaccinated, with two major modifications. First, teachers who are fully vaccinated do not need to enter quarantine after close contact with an infected person (CDC 2021c). In Equation (8), this means that  $Q_{in} = Q_{vir} = Q_{end} = 0$ . Second, the daily probability of receiving a positive COVID-19 test is lower by the vaccine's rate of effectiveness. In Equation (8), this means that  $I$  should be replaced by  $(1 - e)I$ . As a result, **among teachers who are fully vaccinated, the probability of having to stay home at the beginning of day 15 is**

$$(9) \quad U_{vac} = 1 - (1 - (1 - e)I)^{10}.$$

## C. Forecasts for the full group of teachers in a district or school

Among the full group of teachers in a district or school, the forecasted fraction of teachers who will need to stay at home on a typical day is

$$(10) \quad U_{overall} = (1 - v)U_{notvac} + vU_{vac}.$$

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Please visit the [online tool](#) to obtain a forecast for the percentage of teachers who must stay at home on a typical day. This technical supplement was prepared by Hanley Chiang of Mathematica.